

## Section 9.5: Integral Computations with Parametric Curves

**Recall:** In Calculus I, the following formulas were introduced:

- Area under the curve:

$$A = \int_a^b y dx$$

- Volume of revolution around  $x$ -axis:

$$V = \pi \int_a^b y^2 dx$$

- Volume of revolution around  $y$ -axis:

$$V = 2\pi \int_a^b xy dx$$

- Arc length:

$$s = \int_a^b \sqrt{1 + [dy/dx]^2} dx$$

- Area of the surface of revolution around  $x$ -axis:

$$S = 2\pi \int_a^b y \sqrt{1 + [dy/dx]^2} dx$$

- Area of the surface of revolution around  $y$ -axis:

$$S = 2\pi \int_a^b x \sqrt{1 + [dy/dx]^2} dx$$

If  $x = f(t)$  and  $y = g(t)$ , then we get

$$dx = f'(t)dt$$

$$dy = g'(t)dt$$

$$\sqrt{1 + [dy/dx]^2} dx = \sqrt{1 + [g'(t)/f'(t)]^2} f'(t)dt = \sqrt{f'(t)^2 + g'(t)^2} dt$$

By making the appropriate substitutions, we can find area, volume of revolution, arc length, and surface area of smooth parametric curves.

### Important:

1. To evaluate integrals involving area or volume of revolution (which involve  $dx$ ), we integrate either from  $t = \alpha$  to  $t = \beta$  or from  $t = \beta$  to  $t = \alpha$ ; the proper choice of limits on  $t$  being the one that corresponds to traversing the curve in the positive  $x$ -direction from left to right.

2. On all of the integrals, we need to make sure that the curve is traced out only once of the interval of integration.

**Example:**

- (a) Find the area of the region that lies between the given parametric curves and the  $x$ -axis.

$$x = e^{3t}, y = e^{-t}; 0 \leq t \leq \ln 2$$

- (b) Find the volume obtained by revolving around the  $x$ -axis the region described in part (a).

(c) Find the arc length of the given curve.

$$x = r \cos t, \quad y = r \sin t; \quad 0 \leq t \leq 2\pi$$

(d) Find the area of the surface of revolution generated by revolving the region determined by part (c) about the  $x$ -axis. Consider just  $0 \leq t \leq \pi$ .