

### Section 4.5: Summary of Curve Sketching (Part 2)

**Question:** If  $f(x) = p(x)/q(x)$  is a rational function such that  $p(x)$  and  $q(x)$  have no factors in common (i.e., the “fraction” is reduced), then when will  $f(x)$  have a horizontal asymptote? When will it not?

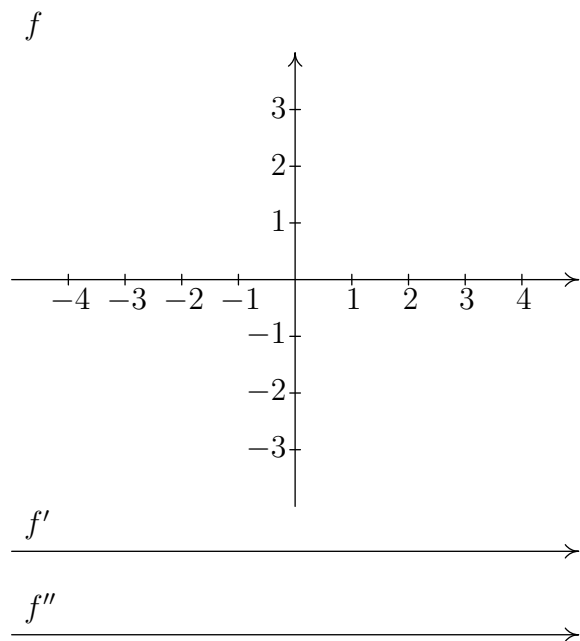
**Answer:**

When the degree of the numerator is \_\_\_\_\_ than the degree of the denominator, other kinds of asymptotes are possible: *slant* (sometimes called *curvilinear* or *oblique*). To see what these new kinds of asymptotes are, we use polynomial long division.

**Theorem:** A rational function cannot have both a horizontal asymptote and a curvilinear (including slant) asymptote. Why?

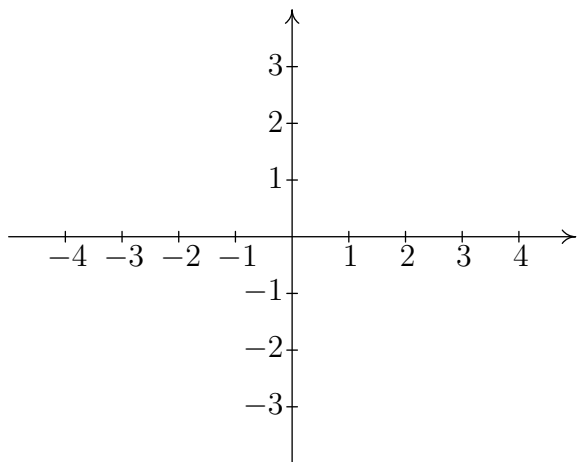
**Example 1:** Identify the curvilinear asymptote of the following function and sketch its graph.

$$g(x) = \frac{x^3 - x^2 + 4}{x - 1}$$

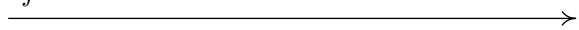


**Example 2:** Sketch the graph of the following function.  $f(x) = \frac{x^2 + x + 1}{x - 1}$

$f$



$f'$



$f''$

