



## MA 2550: Calculus I

### Review for Exam 3

Exam 3 will be on **Tuesday, November 25**. Exam 3 covers material from sections 4.3, 4.4, 4.5, 4.7, 4.9, 5.1, 5.2, 5.3, and 5.4. Material covered on the previous exams is also fair game. In fact, one of the questions on Exam 3 will be identical or nearly identical to a question on Exam 1 or Exam 2 (so, study Exam 1 and Exam 2!). This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to the questions on this review. I will not collect this review and you may do what you want with it.

### TOPICS

Here is a list of topics/concepts that you must have an understanding of or be able to do:

- Understand relationship between the sign of the derivative and increasing/decreasing
- Understand relationship between the sign of the second derivative and concavity
- Know statement of First Derivative Test and be able to use it to make conclusions about local max/min
- know statement of Second Derivative Test and be able to use it to make conclusions about local max/min; when is this test inconclusive?
- Know definition of a point of inflection and be able to find them
- Be able to compute limits at infinity; know shortcuts for rational functions
- Be able to find vertical, horizontal, and slant asymptotes
- Be able to sketch the graph of a function as in the problems we did in Section 4.5; not by copying a picture off your calculator. In reality, I will probably give you a bunch of information about a function and then ask you to sketch the graph from that information. (A great example to look at is the first example I did in class from the supplementary notes from Section 4.5.)
- Be able to solve optimization problems
- Be able to find antiderivatives and evaluate indefinite integrals
- Be able to approximate the area (or net signed area) under the graph of a function
- Be able to approximate a definite integral of a function over an interval
- Know the summation formulas on the top of page 303
- Be able to compute the exact value of net signed area under the graph of a function by taking limits of Riemann sums (using right endpoints)
- Be able to compute the exact value of a definite integral of a function over an interval by taking limits of Riemann sums (using right endpoints)
- Know statements of both parts of the Fundamental Theorem of Calculus

- Be able to compute derivatives of functions defined in terms of integrals
- Be able to compute definite integrals of functions using Part 2 of the Fundamental Theorem of Calculus

## WORDS OF ADVICE

Here are some things to keep in mind when taking the exam:

- Show all work! The thought process and your ability to show how and why you arrived at your answer is more important to me than the answer itself. For example, if you have the right answer, but your reasoning is flawed, then you will lose most of the points.
- The exam will be designed so that you could complete it without a graphing calculator. If you find yourself using your calculator a lot on a given question, then you may be doing something wrong.
- Make sure you have answered the question that you were asked. Also, ask yourself if your answer makes sense.
- If you know you made a mistake, but you can't find it, explain to me why you think you made a mistake and tell me where the mistake might be. This shows that you have a good understanding of the problem.
- If you write down an "=" sign, then you better be sure that the two expressions on either side are equal. Similarly, if two things are equal and it is necessary that they be equal to make your conclusion, then you better use "=".
- Both of us should be able to read what you wrote. Your work should be neat and organized! In general, your work should flow from left to right and then top to bottom (just like if you were reading). Don't make me wander around the page trying to follow your work.
- If your answer is not an entire paragraph (and sometimes it may be), then your answer should be clearly marked.
- Ask questions when you are confused. I will not give away answers, but if you are confused about the wording of a question or whether you have shown sufficient work, then ask me.

## EXERCISES

Try some of these problems. There are a lot of problems below and you don't necessarily need to do all of them. You should do the ones that you think you need more practice on. I'm hoping that you will talk amongst each other to determine if you are doing them correctly. Of course, if you have questions, then I will answer them. Lastly, if a concept appears in multiple questions, you should not necessarily take that to mean that that concept is somehow more important than ones that do not appear frequently.

1. True or False? Justify your answer.
  - (a) If  $f$  is increasing and differentiable on  $(a, b)$ , then  $f'(x) > 0$  for all  $x$  in  $(a, b)$ .
  - (b) If  $f'(c) = 0$ , then  $f$  has either a local minimum or local maximum at  $x = c$ .
  - (c) If  $f$  has an inflection point at  $x = c$ , then  $f''(c) = 0$ .

- (d) Every rational function has an asymptote that is a polynomial function. (Hint: Don't forget that lines, including horizontal ones are polynomials.)
- (e) If a function  $f$  has a vertical asymptote at  $x = a$ , then the graph of  $f$  cannot cross the line  $x = a$ .
- (f) If a function  $f$  has a horizontal asymptote at  $y = b$ , then the graph of  $f$  cannot cross the line  $x = b$ .
2. Provide an example of a function  $f$  such that  $f'(2) = 0$ , but  $f$  does not have a local maximum or local minimum at  $x = 2$ .
3. Provide an example of a function  $f$  such that  $f''(1) = 0$ , but  $f$  does not have an inflection point at  $x = 1$ .
4. Assume that a continuous function  $f$  has exactly one critical number:  $f'(2) = 0$ . In each case below, decide whether the point at which  $x = 2$  is a local max, local min, or neither.
- (a)  $f'(0) = -2$  and  $f'(3) = 2$
- (b)  $f''(2) = -4$
- (c)  $\lim_{x \rightarrow \infty} f(x) = \infty$  and  $\lim_{x \rightarrow -\infty} f(x) = -\infty$
5. Determine the intervals on which the following functions are increasing, decreasing, concave up, and concave down. Also, find the coordinates of all local maximums, local minimums, and inflection points.
- (a)  $f(x) = (x + 1)^3(x - 2)$
- (b)  $g(x) = \cos^2 x$  on interval  $(0, \pi)$
6. Evaluate each of the following limits.
- (a)  $\lim_{x \rightarrow \infty} (-3x^3 + 5x^2 - 3x + 17)$
- (b)  $\lim_{x \rightarrow \infty} \frac{-3x^3 + 5x^2 - 3x + 17}{1 - 2x^3}$
- (c)  $\lim_{x \rightarrow \infty} \frac{-3x^3 + 5x^2 - 3x + 17}{2x^4 + 5x^2 - 4}$
- (d)  $\lim_{x \rightarrow \infty} \frac{-3x^3 + 5x^2 - 3x + 17}{2x^2 + 4x - 6}$
- (e)  $\lim_{x \rightarrow -\infty} \frac{-3x^3 + 5x^2 - 3x + 17}{2x^2 + 4x - 6}$
7. Find *all* asymptotes for each of the following functions.
- (a)  $g(x) = \frac{x}{x^2 + 4}$
- (b)  $h(x) = \frac{\sqrt{x^2 + 1}}{x}$
- (c)  $f(x) = \frac{x^2 + 3x - 2}{x + 2}$
8. Sketch the graph of each of the following functions by finding  $x$ -intercepts,  $y$ -intercepts, vertical asymptotes, horizontal or slant asymptotes, intervals of increase and decrease, coordinates of turning points (local max/min), intervals of concavity, coordinates of inflection points.

(a)  $f(x) = 8x^3 - 2x^4$

(b)  $g(x) = \frac{x^2}{x^2 + 3}$

9. Find two numbers whose sum is 23 and whose product is a maximum.
10. Find the point on the graph of  $f(x) = \sqrt{x+1}$  that is closest to the point  $(3, 0)$ .
11. Exercise 11, page 263.
12. Explain the difference between an indefinite integral and a definite integral.
13. Explain why the Power Rule for Integration,  $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ , does not apply to the function  $f(x) = \frac{1}{x}$ .
14. Find  $f$  such that  $f'(x) = \sqrt{x}$  and  $f(4) = 0$ .
15. Approximate the area under the graph of  $f(x) = \cos^2 x$  on the interval  $[0, \pi]$  by forming 4 rectangles and using right endpoints to determine the height of the rectangles.
16. Evaluate the following definite integrals by taking the limit of Riemann sums. (Use  $x_i^* = x_i$ ; that is, use right endpoints.)

(a)  $\int_0^1 x^2 dx$

(b)  $\int_{-1}^3 2x - 1 dx$

17. Write  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{3k}{n}\right)^2 \frac{3}{n}$  as a definite integral. Do not evaluate the integral.

18. Find  $\int_1^{11} f(x) dx$  if  $\int_0^1 f(x) dx = -7$  and  $\int_0^{11} f(x) dx = 29$ .

19. Evaluate each of the following indefinite or definite integrals.

(a)  $\int 4x^2 - 5x + 3 dx$

(b)  $\int \frac{\cos^3}{1 - \sin^2 x} dx$

(c)  $\int \frac{4 + 5x^{3/2}}{\sqrt{x}} dx$

(d)  $\int_{-2}^5 6 dx$

(e)  $\int_1^4 5 - 2x + 3x^2 dx$

(f)  $\int_0^\pi 3 \sin x dx$

(g)  $\int_0^{\pi/4} \sec \theta \tan \theta d\theta$

20. Find the derivative of each of the following functions.

(a)  $f(x) = \int_0^x \sin t^2 dt$

(b)  $g(x) = \int_1^{x^2} (t^2 + 1)^3 dt$

21. Draw a picture that represents the function  $f(x) = \int_0^x \sin t^2 dt$ .