Plymouth State UNIVERSITY

MA 3110: Logic, Proof, and Axiomatic Systems (Fall 2008)

EXAM 1 (Take-home portion)

NAME:

Instructions: Prove any THREE of the following theorems. If you turn in more than three proofs, I will only grade the first three that I see. I expect your proofs to be well-written, neat, and organized. You should write in complete sentences. Do not turn in rough drafts. What you turn in should be the "polished" version of potentially several drafts.

This portion of Exam 1 is worth 30 points, where each proof is worth 10 points.

These are the simple rules for this portion of the exam:

1. You are NOT allowed to copy someone else's work.
2. You are NOT allowed to let someone else copy your work.
3. You are allowed to discuss the problems with each other and critique each other's work.

If these simple rules are broken, then the remaining exams will be all in-class with no reduction in their difficulty.

This half of Exam 1 is due to my office (Hyde 312) by **2 pm on Friday, October 3rd** (no exceptions). You should turn in this cover page and the three proofs that you have decided to submit.

Good luck and have fun!

Theorem 1: For all $x, y \in \mathbb{R}$, $|x + y| \leq |x| + |y|$.

Note: For this theorem, you may use the fact that $|xy| \leq |x||y|$.

Theorem 2: For all $n, m \in \mathbb{N}$, $n + 1$ divides m and m divides $m + 3$ iff $n = 2$ and $m = 3$.

Theorem 3: If every even natural number greater than 2 is the sum of two primes, then every odd natural number greater than 5 is the sum of three primes.

Theorem 4: There is no smallest positive real number.

Theorem 5: For all rational numbers x and y, if $x < y$, then there exists a rational number r such that $x < r < y$.