MA 3110: Logic and Proof
Study Guidelines for Final Exam

The Final Exam is cumulative. However, an emphasis will be placed on the material covered since the Midterm Exam (Sections 5.1–5.3, 5.5, 7.1–7.4). The exam will consist of two parts: an in-class part and a take-home part.

Part I: In-class exam

The in-class part of Final Exam will take place at 2:30–5:00PM on **Thursday, December 17**. This portion of the exam will test your knowledge of definitions and basic concepts and your ability to construct examples and counterexamples. For each section that we have covered since the Midterm Exam, I have indicated key points that you should address when studying in order to be successful on the in-class portion of the Final Exam.

Section 5.1

- Know the definition of a *function* and the associated notation.
- Be able to determine whether a given relation (as a set of ordered pairs, formula, or "bubble" diagram) determines a function (and be able explain why).
- Know what *domain*, *codomain*, and *range/image* are and be able to identify them for a given function.
- Know the definitions of *1-1*, *onto*, and *1-1 correspondence*.
- Be able to determine whether a given function is 1-1 or onto (and be able explain why) and be able to construction examples (with finite or infinite domain) that are or are not 1-1 and/or onto.

Section 5.2

- Know what the *composition* of two functions is and be able to determine the function that results from composing two functions (with compatible domains and codomains).
- Know statement of Theorem 5.2.3 (the composition of two 1-1, respectively onto, functions is 1-1, respectively onto).
- Know what an *inverse function* is and when a function has an inverse.
- Understand statement of Theorem 5.2.10.

Section 5.3

- Understand what the *inverse image* of a set is and understand the associated notation.
- Be able to determine the inverse image of a given set and given function (e.g., Example 5.3.2 and Exercises 5.3.4 and 5.3.5).

- Understand what the *image* of a set is and understand the associated notation.
- Be able to determine the image of a given set and given function (e.g., Exercise 5.3.8 and 5.3.10).
- Understand statements of Theorem 5.3.6 and 5.3.11 (in particular, notice that one of the statements does not involve equality of sets).
- Part 1 of Problem 5.3.12 is worth thinking about.

Section 5.5

- Know the definition of a *sequence* and understand the associated notation.
- Given a function $f : A \to A$, know the definition of the *iterated map on f based at s_0*.
- Be able to work with and construct examples of iterated maps.
- Know the following definitions associated to sequences:
  - *sequence of distinct terms*
  - *constant sequence*
  - *increasing/decreasing sequence*
  - *monotonic sequence*
  - *bounded from below/above*
- Be able to work with and construct examples of sequences that have various properties.
- Know what a *subsequence* is and understand the associated notation.
- Given a sequence, be able to work with and construct examples of subsequences that have various properties.

Section 7.1

- Know definition of *cardinality*.
- Given two sets A and B, understand what card $A =$ card B, card $A \leq$ card B, and card $A <$ card B mean.
- Be able to determine whether two sets have the same cardinality and be able to construct functions that illustrate the relationship of cardinalities of various sets.

Section 7.2

- Know definitions of *finite* and *infinite*.

Section 7.3

- Know definition of *countable*.
- Know that $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$ are countable.
- Understand significance of Theorem 7.3.5.
- Be able to work with and construct examples of countable sets.

Section 7.4

- Know definition of *uncountable*.
- Know that $(0, 1)$, the set of irrationals, and $\mathbb{R}$ are uncountable.
- Be able to work with examples of uncountable sets (e.g., can you think of two sets A and B, where $A \subseteq B$, A is countably infinite, but B is uncountable?).

Finally, you should be able to call upon your own prodigious mental faculties to respond in flexible, thoughtful, and creative ways to problems that may seem unfamiliar on first glance.

Part 2: Take-home exam

The take-home portion of the Final Exam will consists of six theorems and you will be required to prove any **four** of them. This half of the Final Exam is due by 5:00PM on **Friday, December 18** (no exceptions). These are the simple rules for the take-home portion of the exam:

1. You may freely use any theorems that we have discussed in class or are in the sections of the text that we have covered, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from Theorem 4.3.21, then you should say so.
2. You are NOT allowed to copy someone else's work.
3. You are NOT allowed to let someone else copy your work.
4. You are allowed to discuss the problems with each other and critique each other's work.

I've been very pleased (at least most of the time) with the overall performance of the class. Finish strong! Thanks for a great semester.