Introduction to Linear Algebra, Spring 2007
MATH 3130, Section 001

PROJECT 3

NAME 1:

NAME 2:

NAME 3:

Instructions: In groups of 2 to 3 people (preferably 3), **answer at least one of the first questions below** and then **any other 2 questions**. If you complete more than 3 questions, I will grade the first 3. Each group is required to turn in one set of solutions. Your answers should be written up on separate paper and *neatly presented* (do not write your answers on this sheet). In order to receive full credit on each question, you must justify your answers and show all of your work. When you are using a theorem (or fact in a Blue Box) to make a conclusion, be sure to state which theorem you are using and where you are using it. Use this document as the cover page to your project. The names of the people in your group should appear above. Also, circle the problems below that you have completed. Each problem is worth 10 points. This project is due **Friday, April 27**. Please come talk to me if you have questions. Enjoy!

You must complete at least one of the first 2 questions! Then choose any other 2 questions from the 4 remaining questions.

1. Let V be the set of positive real numbers. Define addition and scalar multiplication in V by

$$x \oplus y = xy$$
$$c \star x = x^c$$

   where the operations on the left are the abstract operations of V, and the operations on the right are the ordinary multiplication and exponentiation of real numbers.

   (a) Show that V is a vector space.

   (b) Find a basis for V and prove that it is a basis.

2. Prove that if A is an $n \times n$ diagonalizable matrix, with all eigenvalues strictly less than 1 in magnitude (i.e., $|\lambda| < 1$), then A^k approaches the zero matrix as $k \to \infty$. *Hint:* A^k approaches the zero matrix if each of the columns of A^k approach $\mathbf{0} \in \mathbb{R}^n$. Consider $A^k\mathbf{e}_i$ (the ith column of A^k).

For the next 3 problems, you'll need to know what a difference equation is. See the definition on page 97.

3. Read Section 4.8 and complete Exercise 22 on page 286.

4. Read Section 4.9 and complete Exercise 4 on page 296.

5. Read Section 5.6 and complete Exercise 4 on page 352.