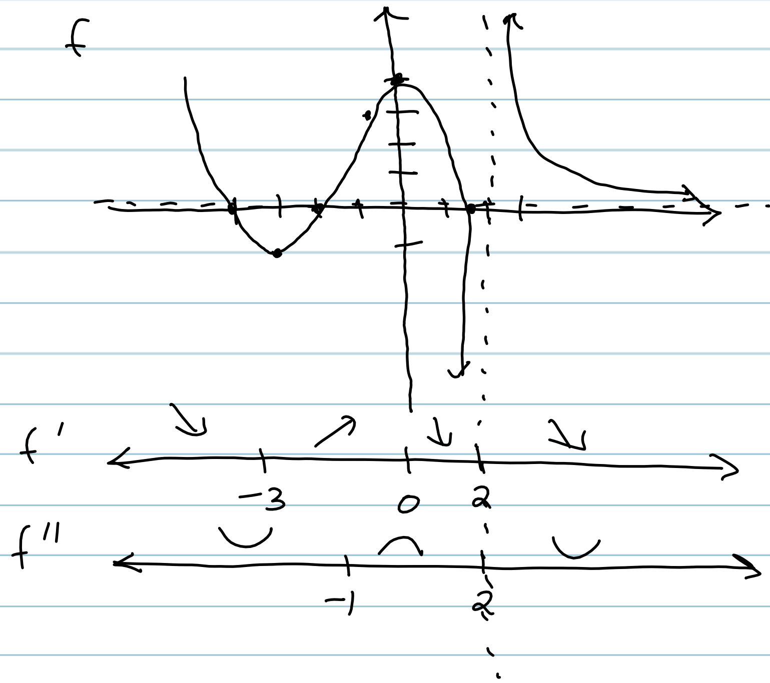
Solns to Exam 3

1.

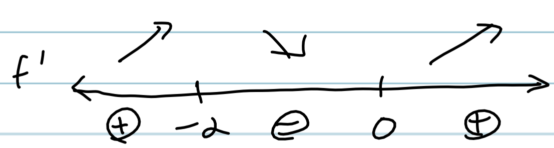


2. $f(x) = 5x^{2/3} + x^{5/3}$

(a) $f'(x) = \frac{10}{3} x^{-1/3} + \frac{5}{3} x^{2/3}$

$$0 = \frac{10 + 5x}{3x^{1/3}} \longrightarrow \boxed{x = -2}, \quad \boxed{x = 0}$$

(b)



$$\boxed{\begin{array}{ll} \text{local max} @ & x = -2 \\ \text{local min} @ & x = 0 \end{array}}$$

3. $f(x) = \frac{x^5}{20} + \frac{-x^4}{6} + \frac{x^3}{6} + 5x + 1$

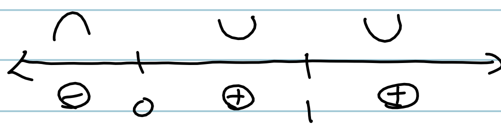
$f'(x) = \frac{5x^4}{20} - \frac{4x^3}{6} + \frac{3x^2}{6} + 5$

$f''(x) = x^3 - 2x^2 + x$

$0 = x(x^2 - 2x + 1)$

$= x \,|\, (x-1) \,|\, (x-1)$

$x = 0 \,|\, x = 1 \,|\, x = 1$



$\boxed{\text{only } x = 0 \text{ is an inflection point}}$

4. $f(x) = \frac{5x^3 - 2x^2 - 1}{x^2 - 4}$

<u>v.a.</u>

$\boxed{x = \pm 2}$

<u>h.a.</u>

$\boxed{\text{none}}$

<u>curvilinear:</u>

$\boxed{y = 5x - 2}$

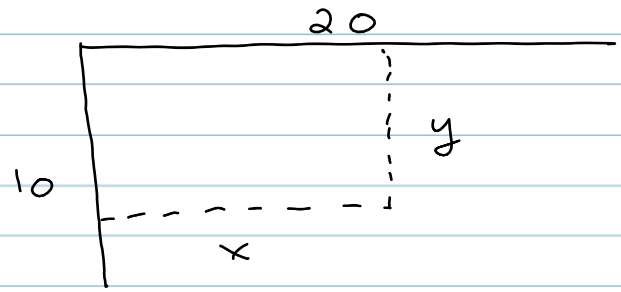
$x^2 - 4 \,|\, 5x^3 - 2x^2 - 1$

$-5x^3 \quad +20x$

$-2x^2 + 20x - 1$

$+2x^2 \quad -8$

5.



(a) $A = xy$

$x + y = 24$

$y = 24 - x$

So, $\boxed{A = x(24-x) \text{ or } 24x - x^2}$

(b) Feasible dom = $\boxed{[14, 20]}$

(c) $A' = 24 - 2x$

$0 = 24 - 2x$

$x = 12$

← but not in feasible dom

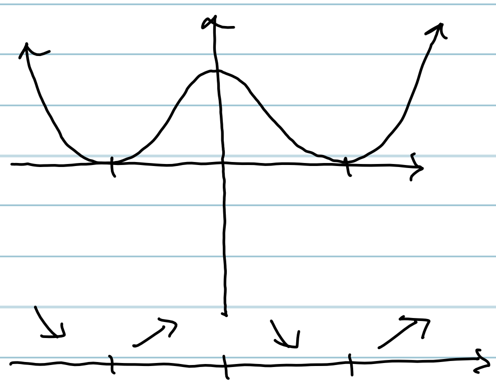
$A(14) = 14(24-14) = 140$ ← larger

$A(20) = 20(24-20) = 80$

So, dims that maximize are

$\boxed{x = 14 \text{ ft} \quad \text{and} \quad y = 10 \text{ ft}}$

6.



7. (a) $\int \frac{4 + x^3}{x^2}\, dx$

$= \int 4x^{-2} + x \, dx$

$= \frac{4x^{-1}}{-1} + \frac{x^2}{2} + C$

$= \boxed{\frac{-4}{x} + \frac{x^2}{2} + C}$

(b) $\int \sec x \,(\tan x + \sec x)\, dx$

$= \int \sec x \tan x + \sec^2 x \, dx$

$= \boxed{\sec x + \tan x + C}$

8. $\int_0^{\pi} \cos^2 x \, dx$

$\approx \frac{\pi}{4}\left[\cos^2\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{2}\right) + \cos^2\left(\frac{3\pi}{4}\right) + \cos^2(\pi)\right]$

$= \frac{\pi}{4}\left[\frac{1}{2} + 0 + \frac{1}{2} + 1\right.$

$= \boxed{\frac{\pi}{2} \text{ units}^2}$

9. $$\int_0^2 x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4i^2}{n^2}\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{8}{n^3} \sum_{i=1}^{n} i^2$$

$$= \lim_{n\to\infty} \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$

$$= \frac{8\cdot 2}{6}$$

$$= \boxed{\frac{8}{3}}$$

10. (a) Many correct ans. Here's one:

$$\boxed{f(x) = x^3}$$

(b) Many correct ans. Here's one:

$$\boxed{g(x) = |x|}$$

11. (Bonus) Let $p(t)$ be position fcn. Then $p(t)$ is cont and diff. By MVT, there exists $c \in [0,2]$ s.t.

$$p'(c) = \frac{p(2) - p(0)}{2 - 0}$$

$$= \frac{158 - 0}{2}$$

$$= 79 \text{ mph.}$$

So, there was @ least one moment in time (that's c) when driver was speeding.