MA 2550: Calculus I (Spring 2009)
Review for Final Exam

The Final Exam is cumulative, which means that any material that we have covered this semester is fair game. Questions on material covered on the first three exams (in-class and take-home) will be similar in nature to the questions asked on those exams. In fact, you may see some of the same questions again. You are also responsible for the material covered since the last exam: Sections 5.2, 5.3, 5.5, 6.1, 6.2, and 6.3. You may bring one **8.5 inch by 11 inch cheat sheet** with you to the exam.

The Final Exam will take place on **Wednesday, May 13 at 8:00–10:30 am**. The Final Exam is worth 150 points.

Topics

To be successful on the material covered since Exam 3, you should

- know statements of both parts of the Fundamental Theorem of Calculus
- be able to compute derivatives of functions defined in terms of integrals
- be able to compute definite integrals of functions using Part 2 of the Fundamental Theorem of Calculus
- be able to evaluate indefinite and definite integrals using u-substitution
- be able to find the area between two curves
- be able to find the volume of a solid of revolution using the Washer Method and the Shell Method

Words of advice

Here are somethings to keep in mind when taking the exam:

- Show all work! The thought process and your ability to show *how* and *why* you arrived at your answer is more important to me than the answer itself. For example, if you have the right answer, but your reasoning is flawed, then you will lose most of the points.
- The exam will be designed so that you could complete it without a graphing calculator. If you find yourself using your calculator a lot on a given question, then you may be doing something wrong.
- Make sure you have answered the question that you were asked. Also, ask yourself if your answer makes sense.
- If you know you made a mistake, but you can't find it, explain to me why you think you made a mistake and tell me where the mistake might be. This shows that you have a good understanding of the problem.
- If you write down an "=" sign, then you better be sure that the two expressions on either side are equal. Similarly, if two things are equal and it is necessary that they be equal to make your conclusion, then you better use "=."
- Don't forget to write limits where they are needed.

- Both of us should be able to read what you wrote. Your work should be neat and organized! In general, your work should flow from left to right and then top to bottom (just like if you were reading). Don't make me wander around the page trying to follow your work.
- If your answer is not an entire paragraph (and sometimes it may be), then your answer should be clearly marked.
- Ask questions when you are confused. I will not give away answers, but if you are confused about the wording of a question or whether you have shown sufficient work, then ask me.

Exercises

Try some of these problems. There are a lot of problems below and you don't necessarily need to do all of them. You should do the ones that you think you need more practice on. I'm hoping that you will talk amongst each other to determine if you are doing them correctly. Of course, if you have questions, then I will answer them. Lastly, if a concept appears in multiple questions, you should not necessarily take that to mean that that concept is somehow more important than ones that do not appear frequently.

1. True or False? Justify your answer.
   (a) $\displaystyle\int_{a/2}^{b/2} f(2x)\ dx = \frac{1}{2}\int_a^b f(x)\ dx$
   (b) Let R be a region in the plane enclosed by some curves. Then the volume of the solid obtained by revolving R about the x-axis is equal to the volume of the solid obtained by revolving R about the y-axis.
2. Find $\displaystyle\int_1^{11} f(x)dx$ if $\displaystyle\int_0^1 f(x)dx = -7$ and $\displaystyle\int_0^{11} f(x)dx = 29$.
3. Evaluate each of the following indefinite or definite integrals.
   (a) $\displaystyle\int 4x^2 - 5x + 3\ dx$
   (b) $\displaystyle\int \frac{\cos^3}{1-\sin^2 x}\ dx$
   (c) $\displaystyle\int \frac{4+5x^{3/2}}{\sqrt{x}}\ dx$
   (d) $\displaystyle\int_{-2}^{5} 6\ dx$
   (e) $\displaystyle\int_1^4 5 - 2x + 3x^2\ dx$
   (f) $\displaystyle\int_0^{\pi} 3\sin x\ dx$
   (g) $\displaystyle\int_0^{\pi/4} \sec\theta\tan\theta\ d\theta$
4. Find the derivative of each of the following functions.
   (a) $f(x) = \displaystyle\int_0^x \sin t^2\ dt$

(b) $g(x) = \int_1^{x^2} (t^2+1)^3 \, dt$

5. Draw a picture that represents the function $f(x) = \int_0^x \sin t^2 \, dt$.

6. Evaluate each of the following integrals.

(a) $\int \frac{x^3+5}{x^2} \, dx$

(b) $\int \frac{x^2}{\sqrt{x^3+5}} \, dx$

(c) $\int \sin^3 x \cos x \, dx$

(d) $\int \sec^2 x \tan x \, dx$

(e) $\int \frac{1}{\sqrt{1-x}} \, dx$

(f) $\int \frac{x}{\sqrt{1-x}} \, dx$

(g) $\int_0^2 x\left(5-x^2\right)^{3/2} \, dx$

(h) $\int_1^9 \frac{\cos\sqrt{x}}{\sqrt{x}} \, dx$

(i) $\int_0^\pi \sin x + \sin 3x \, dx$ (Be careful with your u-sub!)

7. Find the area of the region enclosed by the following graphs.

(a) $y = 12 - x^2$ and $y = x^2 - 6$

(b) $y = \frac{x+4}{x^2}$, $y = 0$, $x = 1$, and $x = 3$

(c) $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \frac{\pi}{2}$

(d) $y = x^3 - x^2 - 2x$ and $y = 4x$

8. The following are two acceleration functions (with respect to time) for two different vehicles.

$$A_1(t) = \sqrt{t} \qquad \text{and} \qquad A_2(t) = \frac{1}{\sqrt{8}}t^2$$

Find the area enclosed by the two acceleration curves and explain what it represents.

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10. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

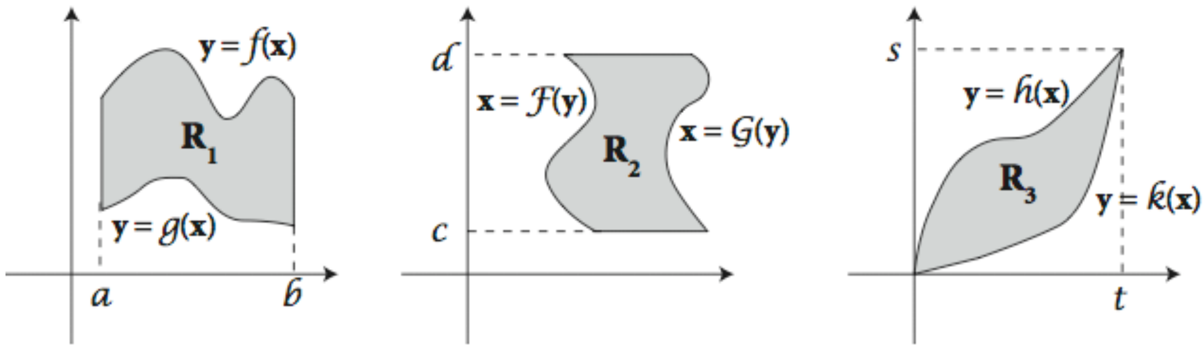
(a) $y = 2 - x^2$, $y = x^2$; about x-axis

(b) $y = x^3$, $y = 4x - x^2$; about y-axis

(c) $f(x) = x$, $x = 1$, $y = 0$; about $y = 3$

(d) $x^2 + y^2 = 1$; about $x = 2$ (a donut!)

11. For each region below, write an expression for the area obtained by rotating the region about the
    (i) x-axis
    (ii) y-axis



12. Let n be a real number greater than 1. Find the volume of the solid formed by revolving the plane region bounded by $x = 1$, $y = \frac{1}{x}$, $x = n$, and $y = 0$ about the x-axis. (Your answer should be in terms of n.) What is the limit of the volume as $n \to \infty$? (If you understand what you just proved, it should blow your mind!)