

Solution to Exercise 11.2.44

$$s = \int_0^{\pi} \sqrt{(dy/dt)^2 + (dx/dt)^2} dt$$

$$= \int_0^{\pi} \sqrt{(-3\sin t + 3\sin 3t)^2 + (3\cos t - 3\cos 3t)^2} dt$$

$$= \int_0^{\pi} \sqrt{9\sin^2 t - 18\sin t \sin 3t + 9\sin^2 3t + 9\cos^2 t - 18\cos t \cos 3t + 9\cos^2 3t} dt$$

$$= \int_0^{\pi} \sqrt{9 + 9 - 18\sin t \sin 3t - 18\cos t \cos 3t} dt$$

$$= \int_0^{\pi} \sqrt{18 - 18(\sin t \sin(t+2t) - \cos t \cos(t+2t))} dt$$

$$= \frac{2\sqrt{3}}{3\sqrt{2}} \int_0^{\pi} \sqrt{1 - \sin t (\sin t \cos 2t + \cos t \sin 2t) - \cos t (\cos t \cos 2t - \sin t \sin 2t)} dt$$

$$= \frac{2\sqrt{3}}{3\sqrt{2}} \int_0^{\pi} \sqrt{1 - \sin^2 t \cos 2t - \cos^2 t \sin 2t - \cos^2 t \cos 2t + \cos^2 t \sin 2t} dt$$

$$= \frac{2\sqrt{3}}{3\sqrt{2}} \int_0^{\pi} \sqrt{1 - \cos 2t (\sin^2 t + \cos^2 t)} dt$$

$$= \frac{2\sqrt{3}}{3\sqrt{2}} \int_0^{\pi} \sqrt{1 - \cos 2t} dt$$

$$= \frac{2\sqrt{3}}{3\sqrt{2}} \int_0^{\pi} \sqrt{1 - (1 - 2\sin^2 t)} dt$$

$$= \frac{2\sqrt{3}}{3\sqrt{2}} \int_0^{\pi} \sqrt{2\sin^2 t} dt$$

$$= \frac{2\sqrt{3}}{3\sqrt{2}} \int_0^{\pi} \sqrt{2} \sin^2 t \, dt$$

$$= \frac{2\sqrt{3}}{3\sqrt{2}} \cdot \sqrt{2} \int_0^{\pi} \sin^2 t \, dt$$

$$= \frac{2\sqrt{6}}{6} \left[-\cos t \right] \Big|_0^{\pi}$$

$$= \frac{2\sqrt{6}}{6} \left[-\cos \pi - (-\cos 0) \right]$$

$$= \frac{2\sqrt{6}}{6} \left[-(-1) + 1 \right]$$

$$= \frac{2\sqrt{6}}{6} \left[2 \right]$$

$$= \boxed{\frac{4\sqrt{6}}{12} \text{ units}}$$