Solns to even problems from 8.1

4. $\int x e^{-x}\,dx$ $\qquad u = x \quad v = -e^{-x}$ $\qquad du = dx \quad dv = e^{-x}\,dx$

$= -xe^{-x} - \int (-e^{-x})\,dx$

$= \boxed{-xe^{-x} - e^{-x} + C}$

10. $\int \sin^{-1} x\,dx$ $\qquad u = \sin^{-1} x \quad v = x$ $\qquad du = \frac{1}{\sqrt{1-x^2}}\,dx \quad dv = dx$

$= x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}\,dx$ $\qquad u = 1 - x^2$, $du = -2x\,dx$, $dx = \frac{du}{-2x}$

$= x\sin^{-1} x - \int \frac{x}{\sqrt{u}} \frac{du}{-2x}$

$= x\sin^{-1} x + \frac{1}{2}\int u^{-1/2}\,du$

$= x\sin^{-1} x + \frac{1}{2} \cdot 2\, u^{1/2} + C$

$= \boxed{x\sin^{-1} x + \sqrt{1-x^2} + C}$

26. $\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) dx$ $\quad u = \arctan\left(\frac{1}{x}\right)$ $\quad v = x$

$du = \frac{\frac{-1}{x^2}}{1+\left(\frac{1}{x}\right)^2} dx$ $\quad dv = dx$

$= x\arctan\left(\frac{1}{x}\right)\Big|_1^{\sqrt{3}} - \int_1^{\sqrt{3}} x \cdot \frac{\frac{-1}{x^2}}{1+\left(\frac{1}{x}\right)^2} dx$

$= x\arctan\left(\frac{1}{x}\right)\Big|_1^{\sqrt{3}} + \int_1^{\sqrt{3}} \frac{x}{1+x^2} dx$ $\quad u = 1+x^2$, $du = 2x\,dx$, $dx = \frac{du}{2x}$

$= x\arctan\left(\frac{1}{x}\right)\Big|_1^{\sqrt{3}} + \int_{x=1}^{x=\sqrt{3}} \frac{x}{u} \cdot \frac{du}{2x}$

$= x\arctan\left(\frac{1}{x}\right)\Big|_1^{\sqrt{3}} + \frac{1}{2}\ln|1+x^2|\Big|_1^{\sqrt{3}}$

$= \sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}\right) + \frac{1}{2}\ln|4| - \arctan 1 - \frac{1}{2}\ln|2|$

$= \boxed{\frac{\pi\sqrt{3}}{3} + \ln 2 - \frac{\pi}{4} - \ln\sqrt{2}}$

$=$ some #