

# MA 3110: Logic and Proof

## Guidelines for Exam 3

Exam 2 covers material in sections 2.3, 2.4, 2.5, 3.1, 3.2, and 3.3 of our textbook, as well as any material discussed in class. The exam will consist of two parts: an in-class part and a take-home part.

### Part I: In-class exam

The in-class part of Exam 3 will take place on **Wednesday, April 29**. This portion of the exam will test your knowledge of definitions and basic concepts. You should be prepared to generate examples and counterexamples. To be successful on the in-class portion of Exam 3 you should

- know definition of a *family of sets*
- understand what an *indexed family of sets* is
- know and understand definitions of *union* and *intersection* over a family of sets (indexed or not)
- develop intuition about the statements in Theorem 2.8
- know the statements of and be able to apply parts (c) and (d) of Theorem 2.9; develop intuition about the remaining parts of this theorem
- know definition of *pairwise disjoint*
- know definition of *inductive set*
- know statements of the PMI and PCI
- be able to write a proof by induction (PMI or PCI) of statements of the form  $(\forall n \in \mathbb{N})P(n)$
- know definition of *Cartesian product* and be able to work with and generate examples
- develop intuition about the statements in Theorem 3.1 (you do not need to memorize this theorem)
- know definition of a *relation* and be able to work with and generate examples
- know the difference between an element of a set and an ordered pair of elements of a set
- be able to identify *domain* and *range* of a relation
- know definition of the *identity relation*
- be able to draw *digraphs* for relations and be able to interpret them
- be able to draw “*bubble*” (*Venn*) *diagrams* for relations and be able to interpret them
- know definition of *inverse relation* and be able to work with inverses

- know definition of the *composition of two relations* and be able to compute one given two relations
- develop intuition about the statements in Theorems 3.2 and 3.3
- know statement of Theorem 3.3(d)
- know definition of an *equivalence relation* on a set  $A$  and be able to show that a given relation is or is not an equivalence relation
- in particular, you should know the definitions of *reflexive*, *symmetric*, and *transitive* and have a working understanding of these concepts
- know definition of *equivalence class* and be able to find them given an equivalence relation
- know the difference between an element of a set and its equivalence class
- have a working understanding of the equivalence relation  $\equiv_m$  on  $\mathbb{Z}$  and the resulting equivalence classes (see Theorem 3.4)
- know definition of *partition*
- know statements of Theorems 3.5 and 3.6 (in particular, you should understand the big picture of this section: given an equivalence relation, we obtain a partition; and given a partition, we can obtain an equivalence relation)
- be able to evaluate the validity of a proposed “proof” of a statement involving relevant definitions
- as well as being able to generate examples, you should be able to construct counterexamples to show that a given statement is false

Finally, you should be able to call upon your own prodigious mental faculties to respond in flexible, thoughtful, and creative ways to problems that may seem unfamiliar on first glance. (Humans are awesome — I don’t care what Doron Zeilberger says.)

## Part 2: Take-home exam

The take-home portion of Exam 3 will consist of 5 theorems and you will be required to prove any 3 of them. This half of Exam 3 is due on **Monday, May 4** and the beginning of class (no exceptions). These are the simple rules for the take-home portion of the exam:

1. You are NOT allowed to copy someone else’s work.
2. You are NOT allowed to let someone else copy your work.
3. You are allowed to discuss the problems with each other and critique each other’s work.

If these simple rules are broken, then the remaining exams will be all in-class with no reduction in their difficulty.